The ML estimate of a parameter

We are studying metapopulations of frogs in small ponds. We assume extinctions occur independently influenced by a suite of variables (i.e., pond size and juxtaposition, and the other usual suspects). We start with a sample of ponds containing frogs and monitor them daily. When the frogs are not longer found in the pond, we note the time this occurs.

We want to estimate the average time required for a pond to go extinct.

We find a pond with no frogs after 10 days. In light of the data, what is our best estimate of the time to extinction θ (and hence, the extinction rate, θ^{-1})

$$y_i = 10 \text{ days}$$

$$L(\theta \mid y_i) = P(y_i \mid \theta)$$

$$P(y_i \mid \theta) = ke^{-ky_i}$$

Take the logs of each side to make expression easier to differentiate:

$$\ln\left[P(y_i \mid k)\right] = \ln(k) - ky_i$$

$$\frac{d\ln\left[P(y_i \mid k)\right]}{dk} = \frac{1}{k} - y_i$$

Set = 0 and solve for *k*:

$$k_{mle} = \frac{1}{y_i} = \frac{1}{10}$$

We now find another pond with no frogs after 18 days. In light of the new data, what is our best estimate of the time to extinction θ (and hence, the extinction rate, θ^{-1})

$$\mathbf{y} = \left[y_i, y_2 \right] = 10 \text{ days, } 18 \text{ days}$$

$$L(\theta \mid \mathbf{y}) = P(y_1 \mid \theta) P(y_2 \mid \theta)$$

$$P(\mathbf{y} \mid k) = k e^{-k y_1} k e^{-k y_2}$$

Take the logs of each side to make expression easier to differentiate:

$$\ln\left[P(\mathbf{y} \mid k) \right] = 2 \ln(k) - k(y_1 + y_2)$$

$$\frac{d\left[P(\mathbf{y} \mid k) \right]}{dk} = \frac{2}{p} - (y_1 + y_2)$$

Set = 0 and solve for k:

$$k_{mle} = \frac{2}{y_1 + y_2} = \frac{2}{10 + 18}$$

Generalizing:

$$k_{mle=} \frac{n}{\sum_{i=1}^{n} y_i}$$