



**BIE5782**

Unidade 7:

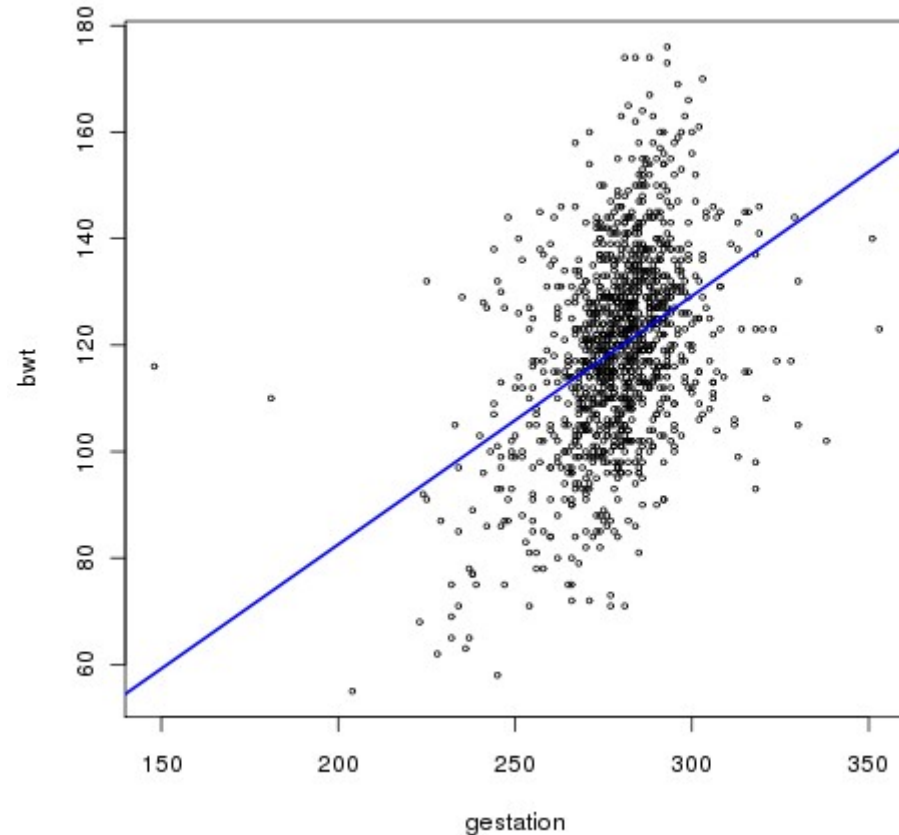
**REGRESSÃO LINEAR  
(SIMPLES)**

# ROTEIRO

1. Motivação
2. Ajuste no R: função  $lm$
3. Resultado no R: objeto  $lm$
4. Método dos mínimos quadrados
5. Premissas, interpretação e diagnóstico

# lm()

## Ajusta Modelo Linear Gaussiano



```
> plot(bwt~gestation, data=babies, cex=0.5)  
> babies.m1 <- lm(bwt~gestation, data=babies)  
> abline(babies.m1, col="blue", lwd=2)
```

# anova.lm()

## Avalia o Modelo

```
> anova(babies.m1)
```

```
Analysis of Variance Table
```

```
Response: bwt
```

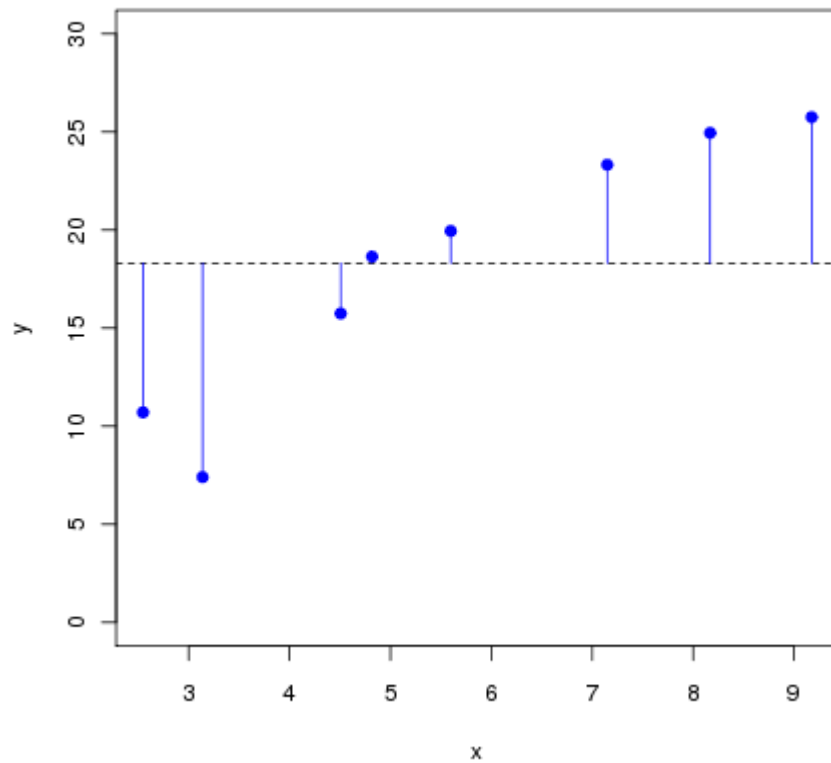
	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
gestation	1	65450	65450	233.43	< 2.2e-16	***
Residuals	1172	328608	280			

```
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```

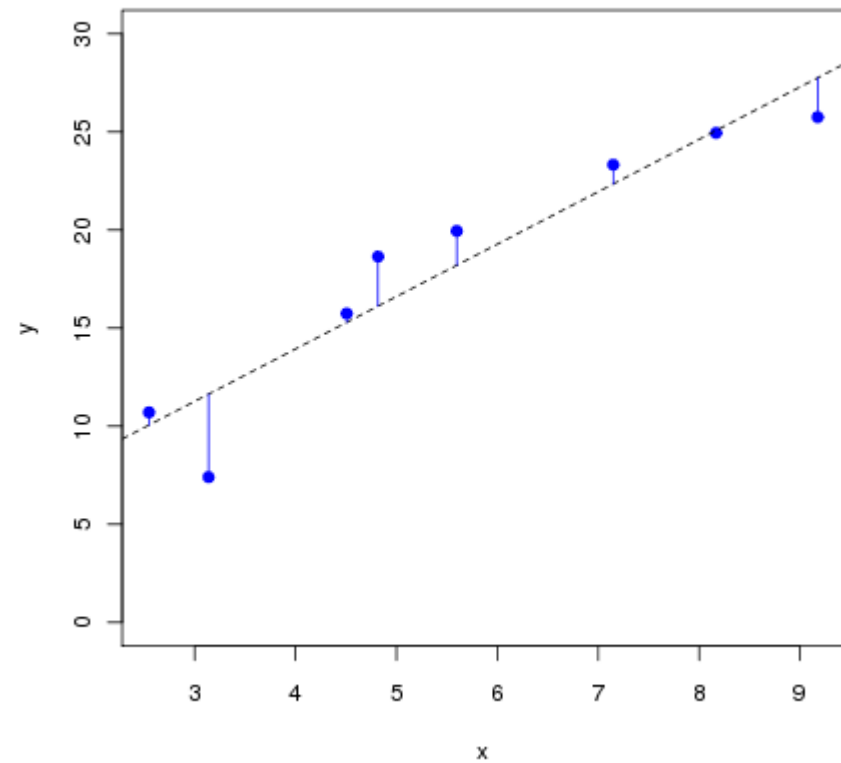
```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'  
0.1 ' ' 1
```

# Somas dos (Desvios) Quadrados

SS Total



SS Erro





# Classe `lm`

```
> names(babies.m1)
[1] "coefficients" "residuals"      "effects"        "rank"
[5] "fitted.values" "assign"         "qr"            "df.residual"
[9] "xlevels"      "call"          "terms"        "model"
```

```
> babies.m1$coefficients
(Intercept)  gestation
-10.7541389   0.4665569
```

```
> babies.m1$residuals[1:4]
      1          2          3          5
-1.748014 -7.814900  8.584770 -12.814900
```

```
> babies.m1$fitted.values[1:4]
      1          2          3          5
121.7480 120.8149 119.4152 120.8149
```

```
> babies.m1$call
lm(formula = bwt ~ gestation, data = babies)
```

Objetos da classe `lm` são listas com todos os objetos resultantes do ajuste de um modelo linear Gaussiano.

**coef(), confint(), residuals(),  
fitted(), logLik(), AIC() ...**

## **Funções de Extração**

```
> coef(babies.m1)
(Intercept)    gestation
-10.7541389     0.4665569

> confint(babies.m1)
                2.5 %      97.5 %
(Intercept) -27.5035066  5.9952288
gestation    0.4066435   0.5264702
```



**coef(), confint(), residuals(),  
fitted(), logLik(), AIC() ...**

## **Funções de Extração**

```
> residuals(babies.m1)[1:4]
      1          2          3          5
-1.748014 -7.814900  8.584770 -12.814900
```

```
> fitted(babies.m1)[1:4]
      1          2          3          5
121.7480 120.8149 119.4152 120.8149
```

```
> logLik(babies.m1) ## pacote MASS
'log Lik.' -4973.256 (df=3)
```

```
> AIC(babies.m1)
[1] 9952.512
```

# summary.lm()

## Resumo do Modelo

```
> summary(babies.m1)
```

Call:

```
lm(formula = bwt ~ gestation, data = babies)
```

Residuals:

Min	1Q	Median	3Q	Max
-49.3483	-11.0653	0.2177	10.1015	57.7037

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-10.75414	8.53693	-1.26	0.208
gestation	0.46656	0.03054	15.28	<2e-16 ***

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 16.74 on 1172 degrees of freedom

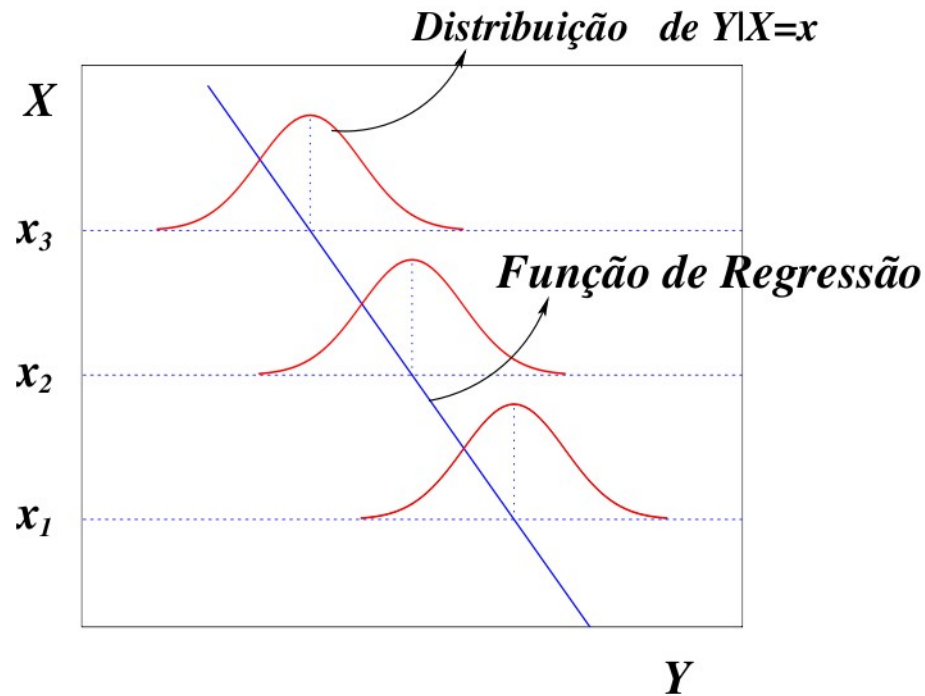
Multiple R-squared: 0.1661, Adjusted R-squared: 0.1654

F-statistic: 233.4 on 1 and 1172 DF, p-value: < 2.2e-16

# O MÉTODO DOS MÍNIMOS QUADRADOS



# Premissas do Modelo de Regressão Linear Gaussiana

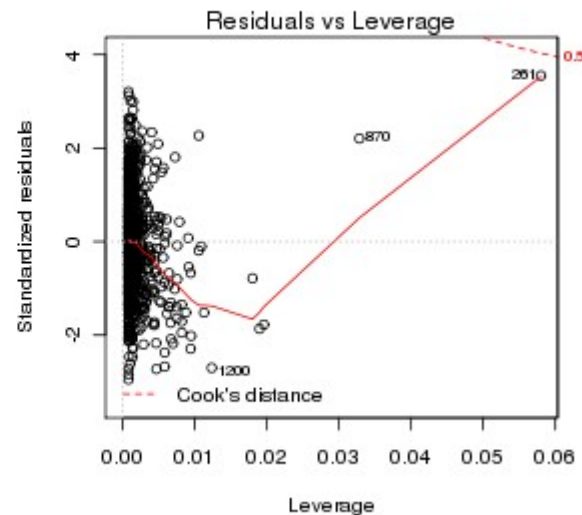
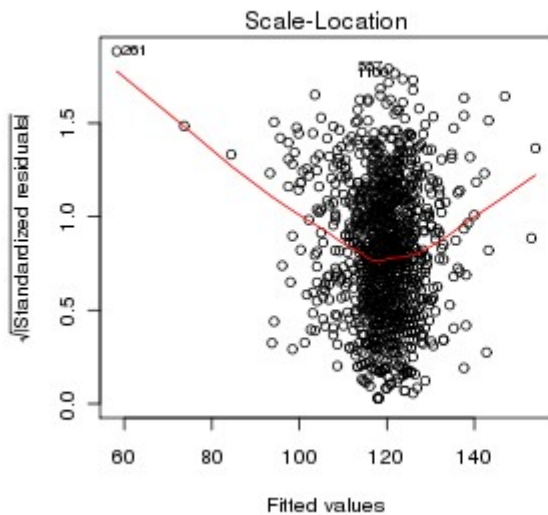
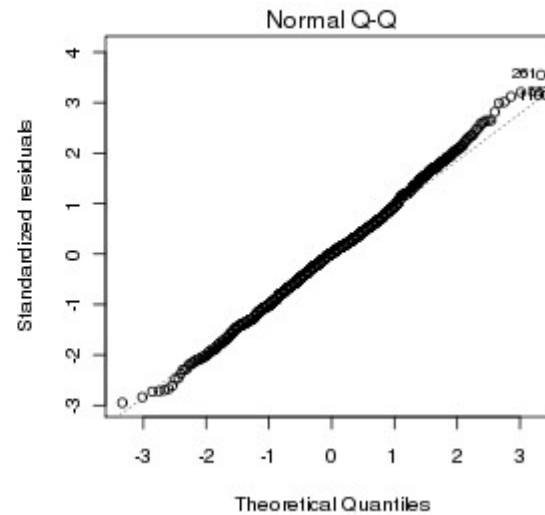
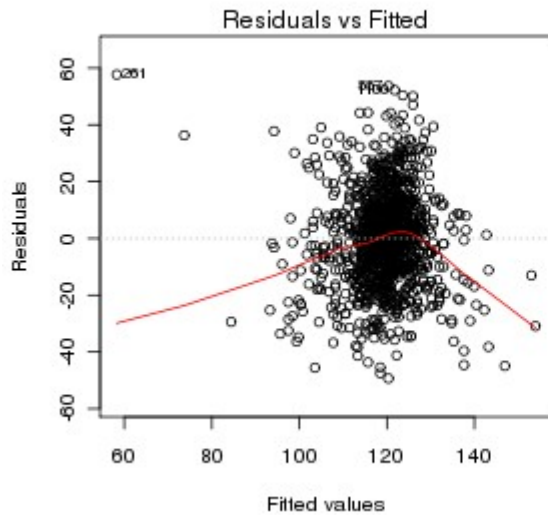


A variável resposta é uma variável normal (Gaussiana) sendo que:

- Sua média é uma função linear das variáveis preditoras;
- Seu desvio-padrão é constante;
- LOGO: resíduos com média zero e variância constante

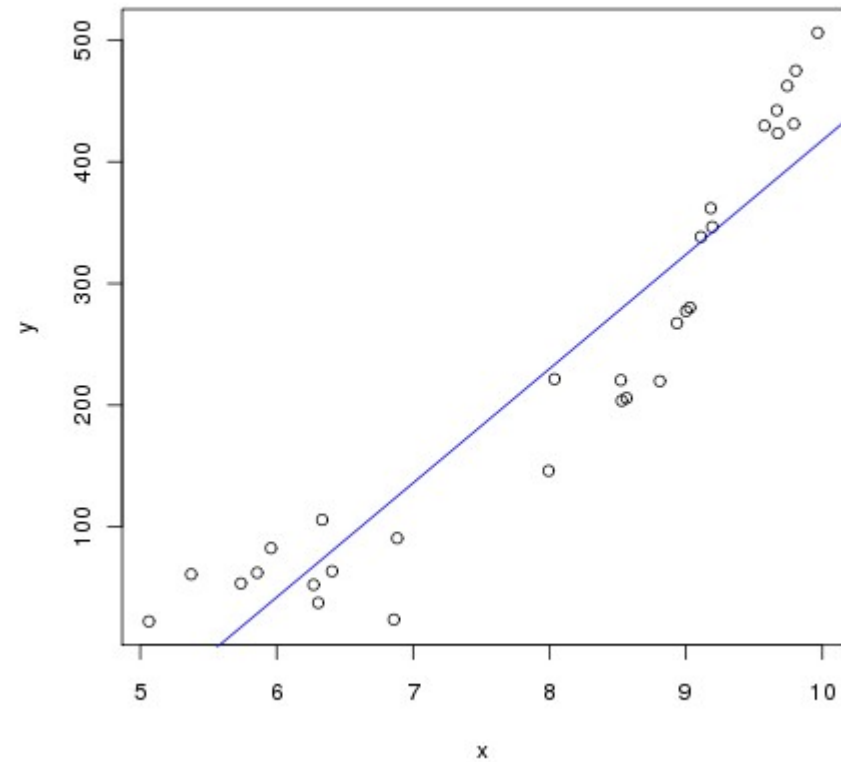
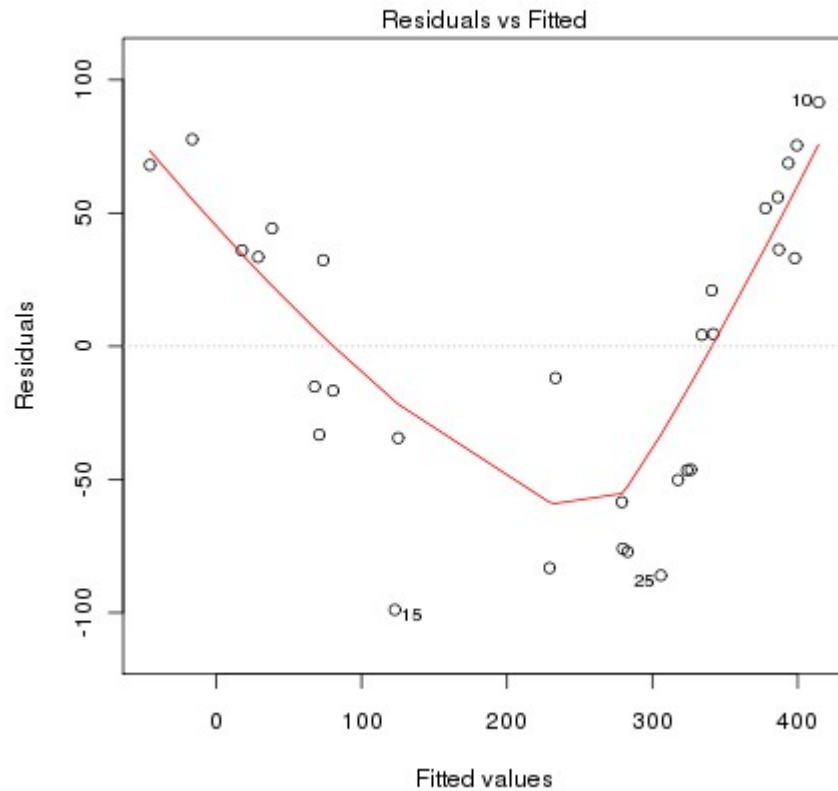
# plot.lm()

## Gráficos de Diagnóstico



```
> par(mfrow=c(2,2))  
> plot(babies.m1)  
> par(mfrow=c(1,1))
```

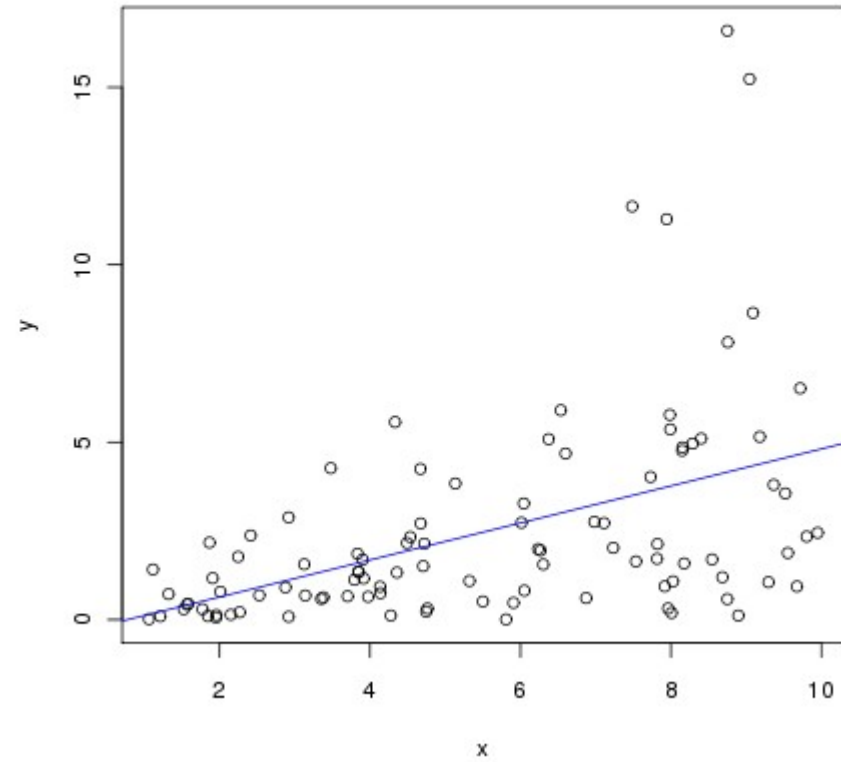
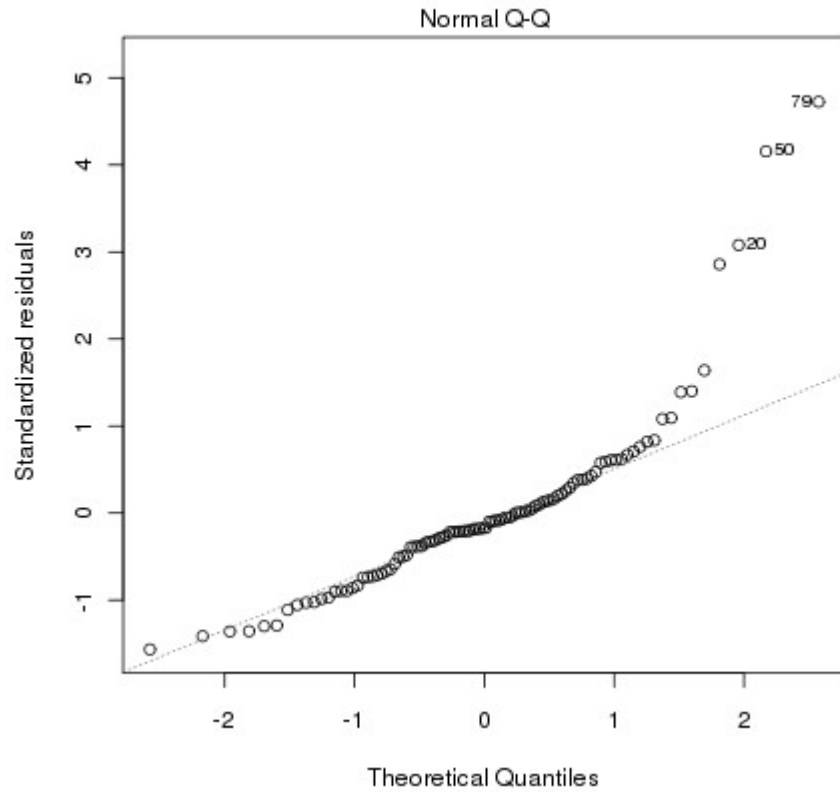
# Resíduos x Estimado



## Detecta:

- Tendências não-lineares
- Variâncias não homogêneas

# Gráfico de Quantis Resíduos x Normal

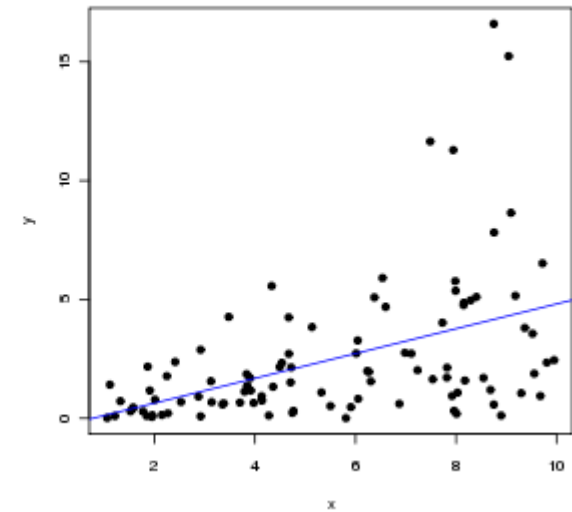
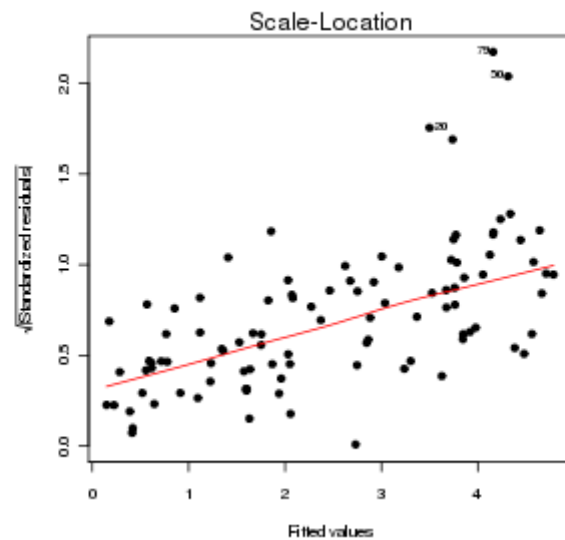
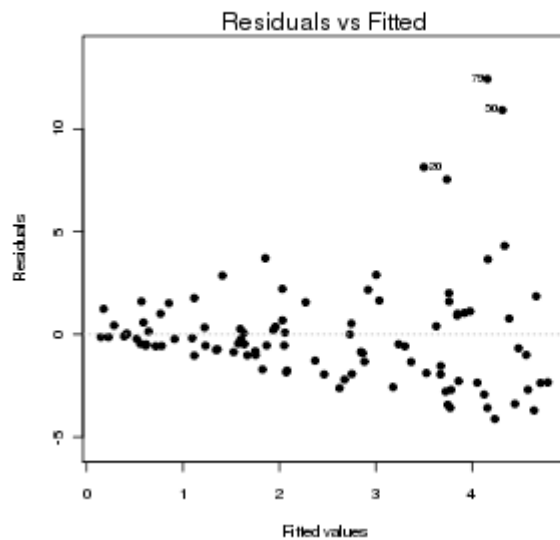


## Detecta:

- Desvios da normalidade nos resíduos

# Resíduos x Estimado

## Raiz dos Resíduos Padronizados x Estimado

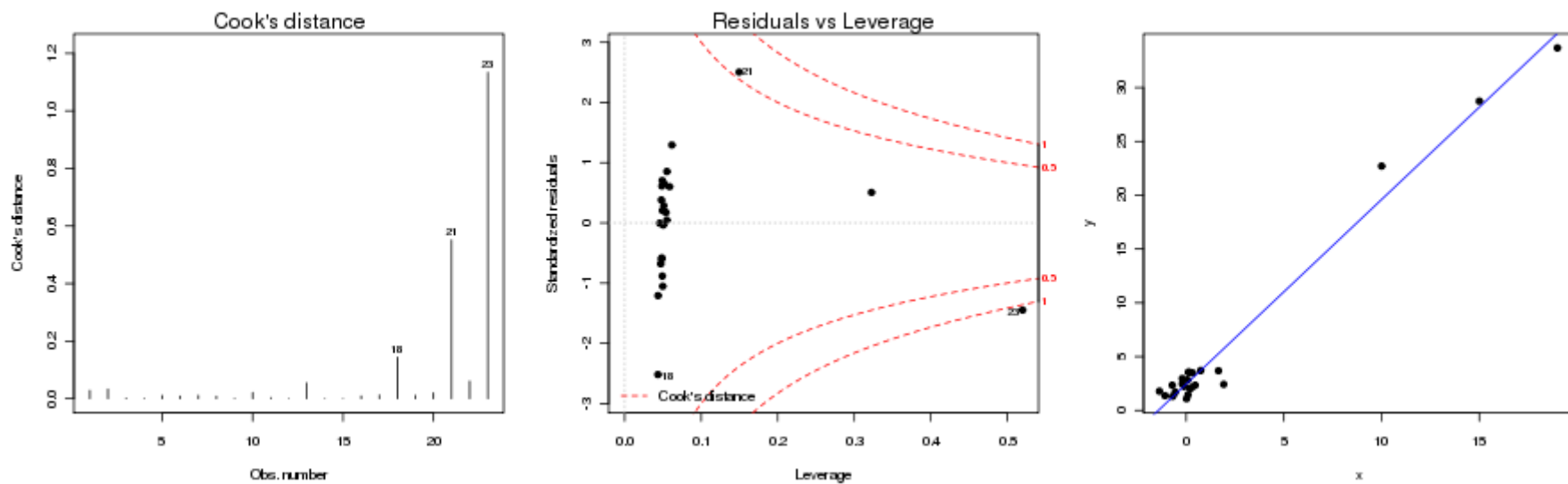


### Detectam:

- Mudanças na variância (heteroscedasticidade);
- Valores extremos não esperados (*outliers*).



# Influência e Alavancagem



Detecta:

- Pontos influentes

# Uma Simulação com Dados Não Normais



# Sugestão de leitura

**John Fox (2002). An R and S-Plus Companion to Applied Regression. Sage Publications, Thousand Oaks, CA, USA.**

# FIM DA PRIMEIRA PARTE



Para a tarde:

Regressão Linear Múltipla